# **Orange Public Schools**

Office of Curriculum & Instruction 2019-2020 Mathematics Curriculum Guide



# Third Grade

Eureka - Module 5: Fractions as Numbers on the Number Line February 10, 2020 – April 3, 2020

Board Approved: 1.14.2020

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# Yearlong Pacing Guide: Third Grade

Eureka Math	Eureka Module Standards
Module 1: Properties of Multiplication and Division and Solving Problems with units of 2-5 and 10 Sept 9- Oct 18	30A1, 30A2, 30A3, 30A4, 30A5, 30A6, 30A7, 30A8
Module 2: Place Value and Problem Solving with Units of Measure Oct 21- Nov 15	3NBT1 , 3NBT2, 3MD1, 3MD2
Module 3:  Multiplication and Division with units of 0, 1, 6-9 and  Multiples of 10  Nov 18- Jan 10	30A3, 30A4, 30A5, 30A7,30A8, 30A9, <mark>3NBT3</mark>
Module 4: Multiplication and Area Jan 13- Feb 7	3.MD.5, 3.MD.6, 3.MD.7
Module 5: Fractions as numbers on the number line Feb 10- April 3	3NF1, 3NF2, 3NF3, 3G2
Module 6: Collecting/ Displaying Data <b>April 6- May 1</b>	3MD3, 3MD4
Module 7: Geometry and Measurement Word Problems <b>May 4- EOSY</b>	<mark>3OA8,</mark> 3MD4, 3MD8, <mark>3G1</mark>

# References

"Eureka Math" Gt Minds. 2018 < <a href="https://greatminds.org/account/product">https://greatminds.org/account/product</a>

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# Module 5 **Essential Questions Enduring Understandings** What do fractions represent? • A region can be divided into equal-sized parts in differ-• What makes fractions equivalent? What is a fracent ways. Equal-sized parts may have the same area, tion? but may not have the same shape. What are different interpretations of a fraction? • A fraction describes the division of a whole (i.e. region, What visual models are most useful when working set, segment) into equal parts. The bottom number of with fractions? a fraction tells how many equal parts the whole is di- What are different ways to compare fractions? vided into. The top number tells how many equal parts What is fraction equivalence and how can it be recare indicated. A fraction is relative to the size of the ognized? • How can whole numbers be expressed as fractions? whole. • Finding a unit-fractional part (1/b) is the same as dividing the whole by the denominator of the fraction. • The set of real numbers is infinite and ordered. Whole number, integers, and fractions are real numbers. Each real number can be associated with a unique point on a number line. • Some points between whole numbers on a number line can be labeled with fractions or mixed numbers. The denominator of the fraction can be determined by counting the number of equal parts between two consecutive whole numbers.

# **Performance Overview**

- Topic A opens Module 5 with students actively splitting different models of wholes into equal parts. They identify and count equal parts as 1 half, 1 fourth, 1 third, 1 sixth, and 1 eighth in unit form before an introduction to the unit fraction 1/b.
- In Topic B, students compare unit fractions and learn to build non-unit fractions with unit fractions as basic building blocks.
- In Topic C, students practice comparing unit fractions with fraction strips, specifying the whole and labeling fractions in relation to the number of equal parts in that whole.
- Students transfer their work to the number line in Topic D. They begin by using the interval from 0 to 1 as the whole. Continuing beyond the first interval, they partition, place, count, and compare fractions on the number line.
- In Topic E, they notice that some fractions with different units are placed at the exact same point on the number line, and therefore are equal. For example, 1/2, 2/4, 3/6, and 4/8 are equivalent fractions. Students recognize that whole numbers can be written as fractions.
- Topic F concludes the module with comparing fractions that have the same numerator. As they compare fractions by reasoning about their size, students understand that fractions with the same numerator and a larger denominator are actually smaller pieces of the whole.

# Module 5: Fractions as Numbers on the Number Line

<u>Pacing:</u>					
	February 10, 2020- April 3, 2020				
	T	Suggested Instructional Days: 31			
Topic	Lesson	Lesson Objective/ Supportive Videos			
Topic A:	Lesson 1	Specify and partition a whole into equal parts, identifying and counting unit fractions using concrete models. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
Partitioning A Whole into Equal Parts	Lesson 2	Specify and partition a whole into equal parts, identifying and counting unit fractions by folding fraction strips. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
	Lesson 3	Specify and partition a whole into equal parts, identifying and counting unit fractions by drawing pictorial area models. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
	Lesson 5	Partition a whole into equal parts and define the equal parts to identify the unit fraction numerically. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
Topic B: Unit Fractions	Lesson 6	Build non-unit fractions less than one whole from unit fractions. https://www.youtube.com/watch?v			
and Their Rela- tion to the Whole	Lesson 7	Identify and represent shaded and non-shaded parts of one whole as fractions. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
	Lesson 8	Represent parts of one whole as fractions with number bonds. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
	Lesson 9	Build and write fractions greater than one whole using unit fractions. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
Topic C: Comparing Unit Fractions and	Lesson 10-11	Compare unit fractions by reasoning about their size using fraction strips.  https://www.youtube.com/watch?v  Compare unit fractions with different sized models representing the whole.  https://www.youtube.com/watch?v			
Specifying the Whole	Lesson 12	Specify the corresponding whole when presented with one equal part.  https://www.youtube.com/watch?v			
	Lesson 13	Identify a shaded fractional part in different ways depending on the designation of the whole. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>			
Mid Module Assessment					

	Lesson 14	Place unit fractions on a number line with endpoints 0 and 1. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
To the D	Lesson 15	Place any fraction on a number line with endpoints 0 and 1. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Topic D: Fractions on the Number Line	Lesson 16	Place whole number fractions and unit fractions between whole numbers on the number Line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 17	Practice placing various fractions on the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 18	Compare fractions and whole numbers on the number line by reasoning about their distance from 0. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 20	Recognize and show that equivalent fractions have the same size, though not necessarily the same shape. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 21	Recognize and show that equivalent fractions refer to the same point on the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 22	Generate simple equivalent fractions by using visual fraction models and the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Topic E: Equivalent Frac-	Lesson 23	Generate simple equivalent fractions by using visual fraction models and the number line. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
tions	Lesson 24	Express whole numbers as fractions and recognize equivalence with different units. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 26	Decompose whole number fractions greater than 1 using whole number equivalence with various models. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 27	Explain equivalence by manipulating units and reasoning about their size. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Topic F:	Lesson 28	Compare fractions with the same numerator pictorially. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Comparison, Or- der, and Size of	Lesson 29	Compare fractions with the same numerator using <, >, or = and use a model to reason about their size. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Fractions	Lesson 30	Partition various wholes precisely into equal parts using a number line method. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	<u> </u>	End Of Module Assessment

Modifications			
Special Education/ 504:	English Language Learners:		
-Adhere to all modifications and health concerns stated in each IEP.  -Give students a menu of options, allowing students to pick assignments from different levels based on difficulty.  -Accommodate Instructional Strategies: reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), handouts, definition list with visuals, extended time  -Allow students to demonstrate understanding of a problem by drawing the picture of the answer and then explaining the reasoning orally and/or in writing, such as Read-Draw-Write  -Provide breaks between tasks, use positive reinforcement, use proximity  -Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum by using manipulatives  -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 17-18)  - Strategies for Students with 504 Plans	<ul> <li>Use manipulatives to promote conceptual understanding and enhance vocabulary usage</li> <li>Provide graphic representations, gestures, drawings, equations, realia, and pictures during all segments of instruction</li> <li>During i-Ready lessons, click on "Español" to hear specific words in Spanish</li> <li>Utilize graphic organizers which are concrete, pictorial ways of constructing knowledge and organizing information</li> <li>Use sentence frames and questioning strategies so that students will explain their thinking/ process of how to solve word problems</li> <li>Utilize program translations (if available) for L1/L2 students</li> <li>Reword questions in simpler language</li> <li>Make use of the ELL Mathematical Language Routines (click here for additional information)</li> <li>Scaffolding instruction for ELL Learners</li> <li>Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 16-17)</li> </ul>		
Gifted and Talented:	Students at Risk for Failure:		
<ul> <li>Elevated contextual complexity</li> <li>Inquiry based or open ended assignments and projects</li> <li>More time to study concepts with greater depth</li> <li>Promote the synthesis of concepts and making real world connections</li> <li>Provide students with enrichment practice that are imbedded in the curriculum such as: <ul> <li>Application / Conceptual Development</li> <li>Are you ready for more?</li> </ul> </li> <li>Common Core Approach to Differentiate Instruction: Students with Disabilities (pg. 20)</li> <li>Provide opportunities for math competitions</li> <li>Alternative instruction pathways available</li> </ul>	- Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum - Modify Instructional Strategies, reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), inclusion of more visuals and manipulatives, Field Trips, Google Expeditions, Peer Support, one on one instruction - Assure constant parental/ guardian contact throughout the year with successes/ challenges - Provide academic contracts to students/ guardians - Create an interactive notebook with samples, key vocabulary words, student goals/ objectives Always plan to address students at risk in your learning tasks, instructions, and directions. Try to anticipate where the needs will be and then address them prior to lessonsCommon Core Approach to Differentiate Instruction: Students with Disabilities (pg 19)		

# 21st Century Life and Career Skills:

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

https://www.state.nj.us/education/cccs/2014/career/9.pdf

- **CRP1**. Act as a responsible and contributing citizen and employee.
- **CRP2**. Apply appropriate academic and technical skills.
- **CRP3**. Attend to personal health and financial well-being.
- **CRP4**. Communicate clearly and effectively and with reason.
- **CRP5**. Consider the environmental, social and economic impacts of decisions.
- **CRP6**. Demonstrate creativity and innovation.

- **CRP7**. Employ valid and reliable research strategies.
- **CRP8**. Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP9**. Model integrity, ethical leadership and effective management.
- **CRP10**. Plan education and career paths aligned to personal goals.
- **CRP11**. Use technology to enhance productivity.
- **CRP12**. Work productively in teams while using cultural global competence.

Students are given an opportunity to communicate with peers effectively, clearly, and with the use of technical language. They are encouraged to reason through experiences that promote critical thinking and emphasize the importance of perseverance. Students are exposed to various mediums of technology, such as digital learning, calculators, and educational websites.

# **Technology Standards:**

All students will be prepared to meet the challenge of a dynamic global society in which they participate, contribute, achieve, and flourish through universal access to people, information, and ideas.

https://www.state.nj.us/education/cccs/2014/tech/

# 8.1 Educational Technology:

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

- A. **Technology Operations and Concepts:** Students demonstrate a sound understanding of technology concepts, systems and operations.
- B. **Creativity and Innovation:** Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
- C. Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
- D. Digital Citizenship: Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
- E. **Research and Information Fluency:** Students apply digital tools to gather, evaluate, and use of information.
- F. Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

# 8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

- A. The Nature of Technology: Creativity and Innovation- Technology systems impact every aspect of the world in which we live.
- B. **Technology and Society:** Knowledge and understanding of human, cultural, and societal values are fundamental when designing technological systems and products in the global society.
- C. **Design:** The design process is a systematic approach to solving problems.
- D. **Abilities in a Technological World:** The designed world in a product of a design process that provides the means to convert resources into products and systems.
- E. **Computational Thinking: Programming-**Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

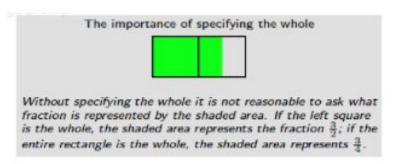
Interdisciplinary Connections:			
English Language Arts:			
RF 3.4 Read with sufficient accuracy and fluency to suppo comprehension.			
W.3.10	Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.		
SL.3.1 Engage effectively in a range of collaborative dissions (one-on-one, in groups, and teacher-led) with diverse partners on <i>grade 3 topics and texts</i> , built on others' ideas and expressing their own clearly.			

# **NJSLS Unpacked Standards**

3.NF.1

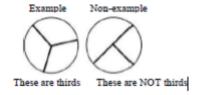
Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

- This standard refers to the sharing of a whole being partitioned. Fraction models in third grade included only are (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a groups) are not addressed in Third Grade.
- In 3.NF. 1 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and reasoning about one part of the whole, e.g. if a whole is partitioned into 4 equal parts then each part is  $\frac{1}{4}$  of the whole, and 4 copies of that part make the whole.
- Students build fractions from unit fractions, seeing the numerator 3 of  $\frac{3}{4}$  as saying that  $\frac{3}{4}$  is the quantity you get by putting 3 pieces of  $\frac{1}{4}$ 's together. There is no need to introduce "improper fraction" initially.



#### Some important concepts related to developing understanding of fractions include:

• Understand fractional parts must be equal-sized

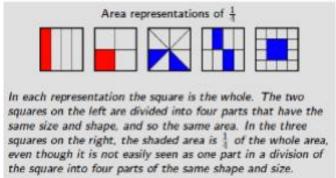


- The number of equal parts tells how many parts make a whole.
- As the number of equal parts in the whole increases, the size of the fractional part decreases.

- The size of the fractional part is relative to the whole. One-half of a small pizza is relatively smaller than one-half of a large pizza.
- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
- Students can count one fourth, two fourths, three fourths.
- Students express fractions as fair sharing or, parts of a whole. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require them to create and reason about fair share.
- Initially, students can use an intuitive notion of "same size and same shape" (congruence) to explain why the parts are equal, e.g., when they divide a square into four equal squares or four equal rectangles. Students come to understand a more precise meaning for "equal parts" as "parts with equal measurements."

#### **Example:**

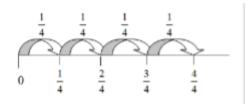
When a ruler is partitioned into halves or quarters of an inch, students see that each subdivision has the same length. In area models students reason about the area of a shaded region to decide what fraction of the whole it represents



# 3.NF.2

Understand a fraction as a number on the number line, represent fractions on a number line diagram.

- a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
- b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number.
- The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that  $\frac{1}{2}$  is between 0 and 1). Students need ample experiences folding linear models (e.g., strings, sentence strips) to help them reason about and justify the location of fractions, such that  $\frac{1}{2}$  lies exactly between 0 and 1.
- In the number line diagram, the space between 0 and 1 is divided (partitioned) into 4 equal parts. The distance from 0 to the first segments is 1 of the 4 parts from 0 to 1 or known as  $\frac{1}{4}$ . Similarly, the distance from 0 to the third segment is 3 parts that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction  $\frac{3}{4}$



3.NF.3

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- b. Recognize and generate simple equivalent fractions, e.g. 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 = 1 at the same point of a number line diagram.
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >,=, or <, and justify the conclusions e.g., by using visual fraction models.
- An important concept when comparing fractions is to <u>look at the size of the parts and the number of the parts</u>. For example,  $\frac{1}{8}$  is smaller than  $\frac{1}{2}$  because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 same size whole is cut into 2 pieces.
- 3.NF.3a and 3.NF.3b: These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures. This standard includes writing whole numbers as fractions. The concepts relates to fractions as division problems, where the fraction <sup>3</sup>/<sub>1</sub> is 3 wholes divided into one group.
- This standard is the building block for later work where students divide set of objects into a specific number of groups. Students understand the meaning of a/1

#### **Example:**

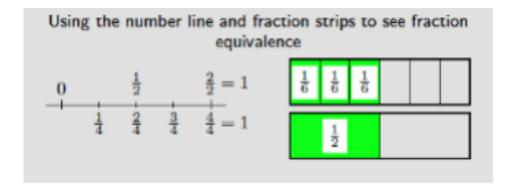
If 6 brownies are shared between 2 people, how many brownies would each person get?

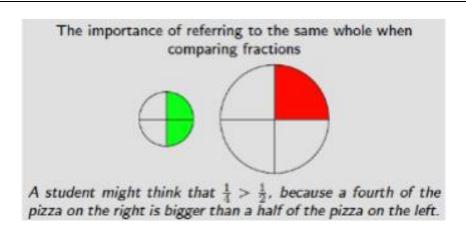
- 3.NF.d: This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, such as  $\frac{1}{3}$  of a cake being larger than  $\frac{1}{4}$  of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.
- In this standard, students should also reason that <u>comparisons are only valid if the wholes are identical.</u> For example,  $\frac{1}{2}$  of a large pizza is a different amount than  $\frac{1}{2}$  of a small pizza. Students should be given opportunities to discuss and reason about which  $\frac{1}{2}$  is larger.
- Previously, in second grade, students compared lengths using a standard measure unit. In third grade, they build
  on this idea to compare fractions with the same denominator. They see that for fractions that have the same
  denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is
  greater because it is made of more unit fractions.

#### **Example:**

A segment from 0 to  $\frac{3}{4}$  is shorter than the segment from 0 to  $\frac{5}{4}$  because it measures 3 segments of  $\frac{1}{4}$  as opposed to 5 segments of  $\frac{1}{4}$ . Therefore,  $\frac{3}{4} < \frac{5}{4}$ .

- Students also see that unit fractions with a larger denominator are smaller, by reasoning that in order for more (identical) pieces to make the same whole, the pieces must be smaller.
- From this idea, they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example,  $\frac{2}{5} > \frac{2}{7}$ , because  $\frac{1}{7} < \frac{1}{5}$ , so 2 pieces of  $\frac{1}{7}$  is less than 2 pieces of  $\frac{1}{5}$ . As with equivalence of fractions, it is important to make sure rgar each fraction refers to the same whole when comparing fractions.



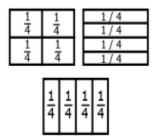


3.G.2

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

• Teacher gives a variety of shapes and students partition them into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.

# Example:



- Help students learn to describe how a shape can be partitioned into four parts with equal area. Talk about the area of each part as  $\frac{1}{4}$  of the area of the shape.
- Students may be confused with the concept that equal shares of identical wholes may not have the same shape. Provide additional experiences about equal shares with different shapes help them understand this confusing concept.

## Common multiplication and division situations. 1

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x? = 18, and 18 ÷ 3 = ?	?x6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?  Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS <sup>2</sup> , AREA <sup>3</sup>	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p + b = ?

<sup>&</sup>lt;sup>1</sup> The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

 $<sup>^2</sup>$  Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

<sup>&</sup>lt;sup>3</sup> The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 5 Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	
Optional Mid –Module Assessment (Interview Style)	3.NF.1 3.NF.3 3.G.2	1 Block	Individual or Small Group with Teacher	
Optional End-of- Module Assessment (Interview Style)	3.NF.1 3.NF.2 3.NF.3 3.G.2	1 Block	Individual or Small Group with Teacher	
Grade 3 Interim 3 Assessment (i-Ready)	3.NF.1 3.NF.2 3.NF.3b 3.NF.3c 3.NF.3d	1 Block	Individual	

Module 5 Performance Assessment/ PBL Framework				
Assessment	Estimated Time	Format		
Module 5 Performance Task 1  Identifying Fractions	3.NF.1	Up to 30 minutes	Individual or Small Group	
Module 5 Performance Task 2  Equivalent Fractions	3.NF.3	Up to 30 minutes	Individual or Small Group	
Extended Constructed Response (ECR)* (click here for access)	Dependent on unit of study & month of administration	Up to 30 Minutes	Individual	

Use the following links to access ECR protocol and district assessment scoring documents:

Assessment and Data in Mathematics Bulletin

ECR Protocol

# Third Grade Ideal Math Block

# Fluency: Whole Group

Sprints, Counting, Whiteboard Exchange

# **Application Problem: Whole Group**

Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Technology Integration: https://embarc.online/

\*Website provides Goggle slides, additional activities, and student videos per lesson

# Concept Development: Individual/partner/whole

Instruction & Strategic Problem Set Questions

Technology Integration: https://embarc.online/

Website provides Goggle slides, additional activities, and student videos. per

lesson

# **Student Debrief: Whole Group**

Exit Ticket: Independent

# **CENTERS/STATIONS:**

# Pairs / Small Group/ Individual

**DIFFERENTIATED** activities designed to **RETEACH**, **REMEDIATE**, **ENRICH** student's understanding of concepts.

M: Meet with the teacher

https://teachertoolbox.com/ A: Application/ Problem Solving T: Technology Resources I-ready Zearn H: Hands on Activities 50-60 min.

20-30 min.

#### **Lesson Structure:**

# Fluency:

- Sprints
- Whiteboard Exchange

Technology Integration:

**Splat Sequences** 

Which one doesn't belong?

Would you rather?

**Esti- Mysteries** 

# **Anchor Task:**

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

# **Guided Practice/ Independent Practice : (**largest chunk of time)

#### Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

#### **Technology Integration:**

#### Think Central:

- Pre-Test
- Chapter Review
- Test Prep
- Performance Tasks

#### https://embarc.online/

Virtual Manipulatives for lessons

http://nlvm.usu.edu/en/nav/vlibrary.html

For videos that students can watch and interact with independently click here

#### **Student Debrief:**

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

#### **Centers:**

- I-Ready: <a href="https://login.i-ready.com/">https://login.i-ready.com/</a> <a href="ji-Ready">ji-Ready</a> makes the promise of differentiated instruction a practical reality for teachers and students. It was designed to get students excited about learning and to support teachers in the challenge of meeting the needs of all learners. Through the power of one intuitive system whose pieces were built from the ground up to work together, teachers have the tools they need to ensure students are on the road to proficiency.
- Zearn: <a href="https://www.zearn.org/">https://www.zearn.org/</a> Zearn Math is a K-5 math curriculum based on Eureka Math with top-rated materials for teacher-led and digital instruction.
- Teacher Toolbox; <a href="https://teacher-toolbox.com/">https://teacher-toolbox.com/</a> A digital collection of K-8 resources to help you differentiate instruction to students performing on, below, and above grade level.

NJSLA Assessment Evidence/Clarification Statements				
NJSLS	Evidence Statement	Clarification	MP	
3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	<ul> <li>Tasks do not involve the number line.</li> <li>Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8</li> </ul>	MP 2	
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	<ul> <li>Fractions may be greater than 1.</li> <li>Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>Fractions equal whole numbers in 20% of these tasks.</li> <li>Tasks have "thin context" 2 or no context.</li> <li>Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> </ul>	MP 5	
3.NF.3a-1	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size	<ul> <li>Tasks do not involve the number line.</li> <li>Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 5	
3.NF.3a-2	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same point on a number line	<ul> <li>Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 5	

3.NF.3b-1	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$ , $4/6 = 2/3$ ).	<ul> <li>Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 7
3.NF.3c	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.	<ul> <li>Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.</li> <li>Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>The explanation aspect of 3.NF.3 is not assessed here.</li> </ul>	MP 3, 5, 7
3.NF.3d	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or	Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) Justifying is not assessed here. For this aspect of 3.NF.3d, see 3.C.3-1 and 3.C.4-4. iv) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.	MP 7
3.NF.A.Int .1	In a contextual situation involving a whole number and two fractions not equal to a whole number, represent all three numbers on a number line diagram, then choose the fraction closest in value to the whole number.	<ul> <li>Fractions equivalent to whole numbers are limited to 0 through 5.</li> <li>Fraction denominators are limited to 2, 3, 4, 6 and 8.</li> </ul>	MP 2,4,5

#### **Number Talks**

#### What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

#### Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

#### **Mental Math**

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

#### **Thumbs Up**

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

#### **Teacher as Recorder**

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

#### **Purposeful Problems**

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

#### **Starting Number Talks in your Classroom**

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

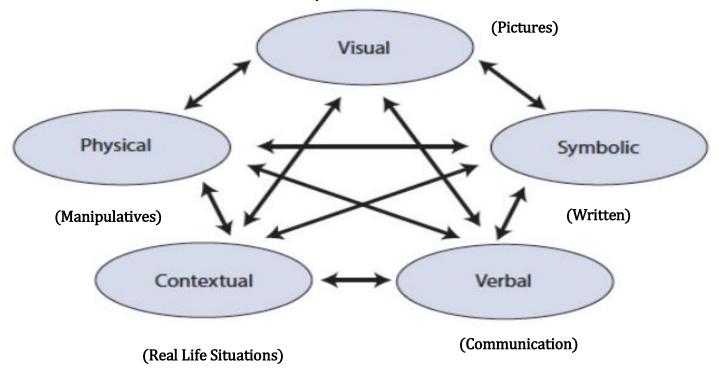
#### The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?

Student Name:	_Task:	School:	Teacher:
Date:			

"I CAN"	STUDENT FRIENDLY RUBRIC				SCORE
	a start 1	getting there 2	that's it 3	WOW!	
Understand	I need help.	I need some help.	I do not need help.	I can help a class- mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think- ing.	

# **Use and Connection of Mathematical Representations**



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical**: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal**: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

**Symbolic**: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

#### The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

# **Concrete Pictorial Abstract (CPA) Instructional Approach**

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

**Concrete:** "Doing Stage": Physical manipulation of objects to solve math problems.

**Pictorial:** "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

#### Read, Draw, Write Process

**READ** the problem. Read it over and over.... And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

# **Mathematical Discourse and Strategic Questioning**

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

#### **Teacher Questioning:**

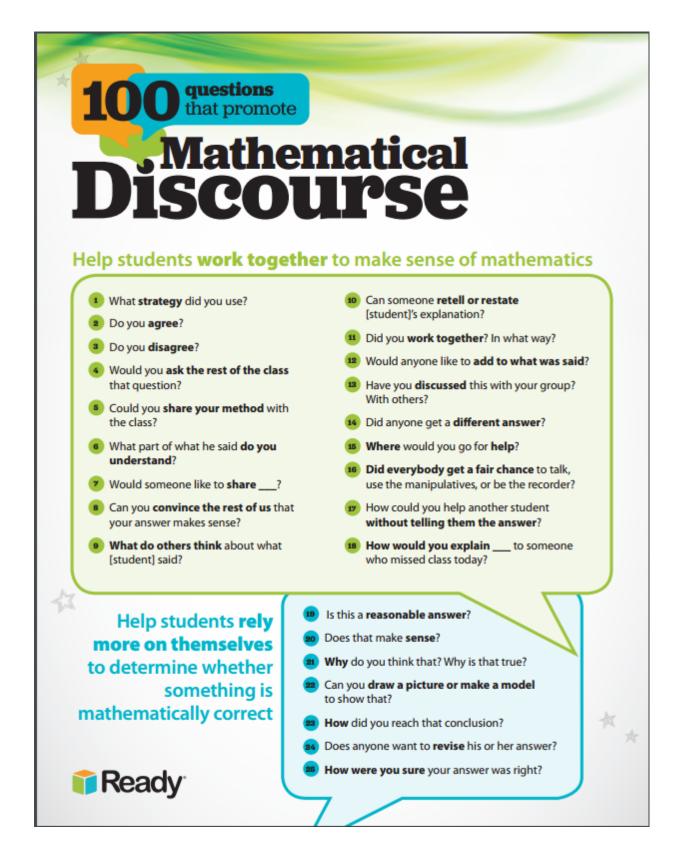
Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?

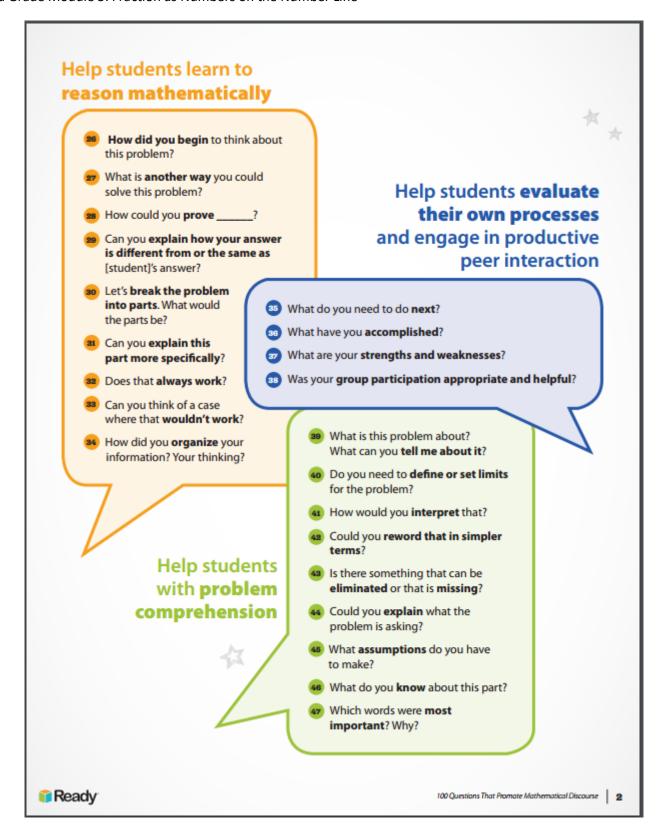


Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

Dr.

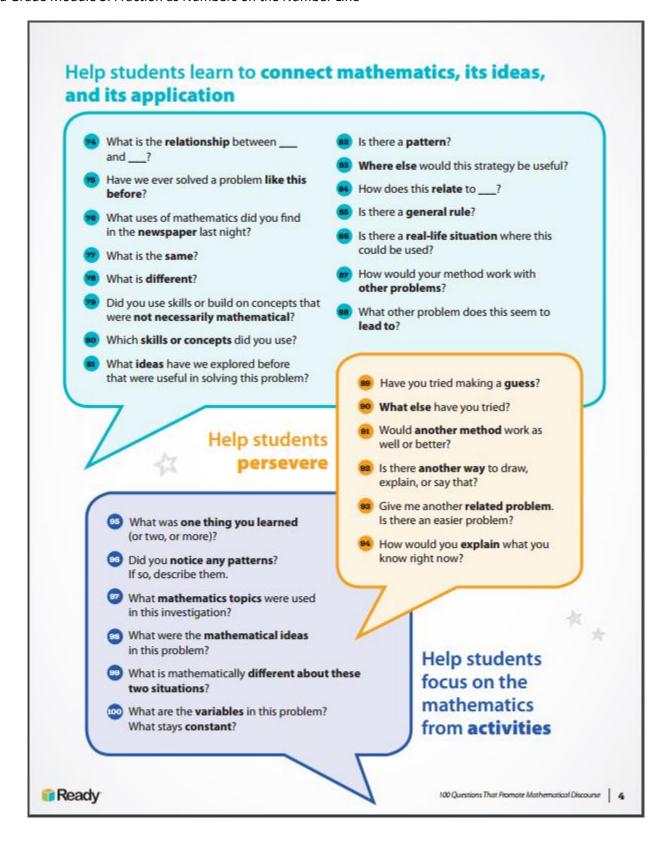




# Help students learn to conjecture, invent, and solve problems What would happen if \_\_\_\_? How would you draw a diagram or make a sketch to solve the problem? Do you see a pattern? Is there another possible answer? What are some possibilities here? If so, explain. 61 Where could you find the information Is there another way to solve the problem? you need? Is there another model you could use to 62 How would you check your steps or solve the problem? your answer? 60 Is there anything you've overlooked? What did not work? How did you think about the problem? 60 How is your solution method the same as or different from [student]'s method? 66 What was your estimate or prediction? Other than retracing your steps, how How confident are you in your answer? can you determine if your answers are What else would you like to know? appropriate? What do you think comes next? 60 How did you organize the information? Do you have a record? Is the solution reasonable, considering the context? How could you solve this using tables, lists, pictures, diagrams, etc.? Did you have a system? Explain it. 68) What have you tried? What steps did Did you have a strategy? Explain it. you take? Did you have a design? Explain it. 69 How would it look if you used this model or these materials?

Ready

100 Questions That Promote Mathematical Discourse 3



#### **Conceptual Understanding**

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

# **Procedural Fluency**

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

## Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the <u>mind</u> with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

#### 3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)

3.NBT.A.2: Add/subtract within 1000

4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm

**5.NBT.B.5:** Multi-digit multiplication/ Use of Standard Algorithm

### **Evidence of Student Thinking**

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

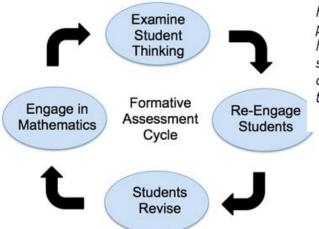
#### **Mathematical Proficiency**

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- <u>Strategic competence</u>: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- <u>Productive disposition</u>: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

#### **Evidence should:**

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

## **Connections to the Mathematical Practices**

## **Student Friendly Connections to the Mathematical Practices**

- 1. I can solve problems without giving up.
- 2. I can think about numbers in many ways.
- 3. I can explain my thinking and try to understand others.
- 4. I can show my work in many ways.
- 5. I can use math tools and tell why I choose them.
- 6. I can work carefully and check my work.
- 7. I can use what I know to solve new problems.
- 8. I can discover and use short cuts.

## The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

## Make sense of problems and persevere in solving them Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways 1 to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers. Reason abstractly and quantitatively Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from 2 whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. Construct viable arguments and critique the reasoning of others In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equa-3 tions. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. Model with mathematics Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, 4 or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

fractions.

## Use appropriate tools strategically Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph pa-5 per or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. Attend to precision As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure 6 and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot. Look for and make use of structure In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate rep-7 resentations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule. Look for and express regularity in repeated reasoning Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine pat-8 terns and generate their own algorithms. For example, students use visual fraction models to write equivalent

# **Effective Mathematics Teaching Practices**

**Establish mathematics goals to focus learning**. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions**. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics**. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

	5 Practices for Orchestrating Productive Mathematics Discussions			
Practice	Description/ Questions			
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task?			
	How do you respond to the work that students are likely to produce?			
	Which strategies from student work will be most useful in addressing the mathematical goals?			
2. Monitoring	Paying attention to what and how students are thinking during the lesson.			
	Students working in pairs or groups			
	Listening to and making note of what students are discussing and the strategies they are using			
	Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)			
3. Selecting	This is the process of deciding the what and the who to focus on during the discussion.			
4. Sequencing	What order will the solutions be shared with the class?			
5. Connecting	Asking the questions that will make the mathematics explicit and understandable.			
	Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.			

## **MATH CENTERS/ WORKSTATIONS**

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

#### **Station Organization and Management Sample**

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

# MATH WORKSTATION INFORMATION CARD

th Workstation:		Time:
SLS.:		
 jective(s): By the end of this task, I wil	l he able to:	
•		
•		
sk(s):		
•		
•		
t Ticket:		
•		
•		

# MATH WORKSTATION SCHEDULE

# Week of:

DAY	Technology	Problem Solving Lab	Fluency	Math	Small Group In-
	Lab		Lab	Journal	struction
Mon.					
	Group	Group	Group	Group	BASED
Tues.					ON CURRENT OB-
	Group	Group	Group	Group	SERVATIONAL DA-
Wed.					TA
	Group	Group	Group	Group	
Thurs.					
	Group	Group	Group	Group	
Fri.					
	Group	Group	Group	Group	

### **INSTRUCTIONAL GROUPING**

	in structure	.,	
	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	

## **Third Grade PLD Rubric**

Go	t It	Tima Grade i Eb Rabile	Not There Yet	
Evidence shows that the stu	dent essentially has the	Student shows evidence of a major misunderstanding, incorrect concepts or proce-		
target concept or big math i	dea.	dure, or a failure to engage in the task.		
PLD Level 5: 100%	PLD Level 4: 89%	PLD Level 3: 79%	PLD Level 2: 69%	PLD Level 1: 59%
Distinguished command	Strong Command	Moderate Command	Partial Command	Little Command
Student work shows <b>dis</b> -	Student work shows	Student work shows mod-	Student work shows par-	Student work shows little
tinguished levels of un-	strong levels of under-	erate levels of under-	tial understanding of the	understanding of the
derstanding of the math-	standing of the mathe-	standing of the mathemat-	mathematics.	mathematics.
ematics.	matics.	ics.		
			Student <b>constructs</b> and	Student attempts to con-
Student constructs and	Student <b>constructs</b> and	Student <b>constructs</b> and	communicates an incom-	structs and communicates
communicates a complete	communicates a com-	communicates a complete	plete response based on	a response using the:
response based on expla-	plete response based on	response based on expla-	student's attempts of ex-	<ul> <li>properties of opera-</li> </ul>
nations/reasoning using	explanations/reasoning	nations/reasoning using	planations/ reasoning us-	tions
the:	using the:	the:	ing the:	<ul> <li>relationship between</li> </ul>
				addition and subtrac-
<ul> <li>properties of opera-</li> </ul>	<ul> <li>properties of opera-</li> </ul>	<ul> <li>properties of opera-</li> </ul>	<ul> <li>properties of opera-</li> </ul>	tion relationship
tions	tions	tions	tions	<ul> <li>Use of math vocabu-</li> </ul>
<ul> <li>relationship between</li> </ul>	<ul> <li>relationship between</li> </ul>	relationship between	<ul> <li>relationship between</li> </ul>	lary
addition and subtrac-	addition and subtrac-	addition and subtrac-	addition and subtrac-	
tion relationship	tion relationship	tion relationship	tion relationship	
<ul> <li>Use of math vocabu-</li> </ul>	<ul> <li>Use of math vocabu-</li> </ul>	Use of math vocabulary	Use of math vocabu-	Response includes <b>limited</b>
lary	lary		lary	evidence of the progres-
		Response includes a logical		sion of mathematical rea-
Response includes an effi-	Response includes a logi-	but incomplete progres-	Response includes an in-	soning and understanding.
cient and logical progres-	cal progression of math-	sion of mathematical rea-	complete or illogical pro-	
sion of mathematical rea-	ematical reasoning and	soning and understanding.	gression of mathematical	
soning and understanding.	understanding.	Contains <b>minor errors</b> .	reasoning and under-	
			standing.	
5 points	4 points	3 points	2 points	1 point

Third Grade Module 5: Fraction as Numbers on the Number Line

### **DATA DRIVEN INSTRUCTION**

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?



Data Analysis Form	School:	Teacher:	Date:	
Assessment:		NJSLS:		
GROUPS (STUDENT INITIALS)	SUPPORT PLAN		PROGRESS	
MASTERED (86% - 100%) (PLD 4/5):				
DEVELOPING (67% - 85%) (PLD 3):				
INSECURE (51%-65%) (PLD 2):				
1113ECOKE (31%-03%) (FED 2).				
BEGINNING (0%-50%) (PLD 1):				

#### **MATH PORTFOLIO EXPECTATIONS**

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the NJSLS. The September task entry(ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

#### **GENERAL PORTFOLIO EXPECTATIONS:**

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual",
   "Partner/Group", and "Individual w/Opportunity for Student Interviews<sup>1</sup>.
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is recommended.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)<sup>2</sup>.
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

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# 4<sup>th</sup> Grade Authentic Assessment #1 – Identifying a Fraction

Name:		Date:	3.NF.1
Kai shaded fo	our pieces of paper with a	a gray crayon.	D
Which piece	e of paper is 3/4 sh	naded?	
Use what y	ou know about frac	tions to explain why your	answer is correct.

## Authentic Assessment Scoring Rubric – Identifying a Fraction

**3.NF.1**: Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

Teacher notes:

Student learning targets for this task may include:

- I can explain any unit fraction as one part of a whole.
- I can explain any fraction (a/b) as "a" (numerator) being the numbers of parts and "b" (denominator) as the total number of equal parts in the whole.
- I can represent a fraction and explain my representation.

One meaning for a fraction is a number that represents a part of the whole. When a fraction is used to describe part of a region, the whole needs to be divided into equal parts. When dividing a region into equal parts, it is not necessary that the parts have the same shape as long as they have the same area.

Students who demonstrate mastery can identify that C shows ¾ shaded. They should also be able to explain that they counted the total number of equal parts to find the denominator and then counted the number of pieces that had been shaded to find the numerator.

Students who demonstrate partial mastery may choose the correct picture, but may not be able to explain how they found the answer.

Level 5: Distinguished	Level 4:	Level 3:	Level 2:	Level 1:
Command	Strong Command	Moderate Command	Partial Command	No Command
Clearly constructs and	Clearly constructs and	Constructs and communi-	Constructs and com-	The student
communicates a complete	communicates a com-	cates a complete response	municates an incom-	shows no work
response based on expla-	plete response based on	based on explana-	plete response based	or justification
nations/reasoning using	explanations/	tions/reasoning using the:	on explana-	-
<ul> <li>Understanding of equal parts</li> <li>Understanding of numerator and denominator</li> </ul>	reasoning using the:  • Understanding of equal parts  • Understanding of numerator and denominator	<ul> <li>Understanding of equal parts</li> <li>Understanding of nu- merator and denomi- nator</li> </ul>	tions/reasoning using the:  • Understanding of equal parts  • Understanding of numerator and denominator	
Response includes an <u>efficient</u> and logical progression of steps.	Response includes a logical progression of steps	Response includes a logical but incomplete progression of steps. Minor calculation errors.	Response includes an incomplete or Illogical progression of steps.	

# 4<sup>th</sup> Grade Authentic Assessment #2- Equivalent Fractions

Mrs. Caha asked her class to write fractions on their whiteboards that were equivalent to  $\frac{1}{2}$ .

Tell if each student's fraction is equivalent to Mrs. Caha's fraction and show how you know.

Gloria: $\frac{3}{4}$	CIRCLE ONE  O Yes  O No	Show how you know:
Isaiah: $\frac{2}{3}$	CIRCLE ONE  O Yes  O No	Show how you know:
Thomas: $\frac{4}{8}$	CIRCLE ONE  O Yes  O No	Show how you know:

# **Authentic Assessment Scoring Rubric –** Equivalent Fractions

<u>3.NF.A.3</u>: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Level 5: Distinguished	Level 4:	Level 3:	Level 2:	Level 1:
Command	Strong Command	Moderate Command	Partial Command	No Command
Clearly constructs and	Clearly constructs and	Constructs and com-	Constructs and	The student
communicates a com-	communicates a com-	municates a complete	communicates an	shows no work
plete response based on	plete response based	response based on ex-	incomplete response	or justification
explanations/reasoning	on explanations/	planations/	based on explana-	
using the:	reasoning using the:	reasoning using the:	tions/ reasoning using the:	
Number line, visual model, reasoning about size  Response includes an efficient and logical progression of steps.	Number line, visual model, reasoning about size  Response includes a logical progression of steps	Number line, visual model, reasoning about size  Response includes a logical but incomplete progression of steps. Minor calculation errors.	Number line, visual model, reasoning about size  Response includes an incomplete or Illogical progression of steps.	

# **Core Instructional and Supplemental Materials (K-5)**

EUREKA MATH v. 2019 (GREAT MINDS)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
<b>K</b> (v. 2019.)	<ul> <li>Teacher Edition: Module 1-6</li> <li>Eureka Math Teacher Resource Pack</li> <li>Eureka K-5 PD Toolkit</li> </ul>	<ul> <li>Learn Workbook Set: Module 1-6</li> <li>Succeed Workbook Set: Module 1-6</li> <li>Practice Workbook, Fluency: Module 1-6</li> </ul>
1	<ul> <li>Teacher Edition: Module 1-6</li> <li>Eureka Math Teacher Resource Pack</li> <li>Eureka K-5 PD Toolkit</li> </ul>	<ul> <li>Learn Workbook Set: Module 1-6</li> <li>Succeed Workbook Set: Module 1-6</li> <li>Practice Workbook, Fluency: Module 1-6</li> </ul>
2	<ul> <li>Teacher Edition: Module 1-8</li> <li>Eureka Math Teacher Resource Pack</li> <li>Eureka K-5 PD Toolkit</li> </ul>	<ul> <li>Learn Workbook Set: Module 1-8</li> <li>Succeed Workbook Set: Module 1-8</li> <li>Practice Workbook, Fluency: Module 1-8</li> </ul>
3		
4	<ul> <li>Teacher Edition: Module 1-7</li> <li>Eureka Math Teacher Resource Pack</li> <li>Eureka K-5 PD Toolkit</li> </ul>	<ul> <li>Learn Workbook Set: Module 1-7</li> <li>Succeed Workbook Set: Module 1-7</li> <li>Practice Workbook, Fluency: Module 1-7</li> </ul>
5	<ul> <li>Teacher Edition: Module 1-7</li> <li>Eureka Math Teacher Resource Pack</li> <li>Eureka K-5 PD Toolkit</li> </ul>	<ul> <li>Learn Workbook Set: Module 1-7</li> <li>Succeed Workbook Set: Module 1-7</li> <li>Practice Workbook, Fluency: Module 1-7</li> </ul>
	<ul> <li>Teacher Edition: Module 1-6</li> <li>Eureka Math Teacher Resource Pack</li> <li>Eureka K-5 PD Toolkit</li> </ul>	<ul> <li>Learn Workbook Set: Module 1-6</li> <li>Succeed Workbook Set: Module 1-6</li> <li>Practice Workbook, Fluency: Module 1-6</li> </ul>

#### MATH IN FOCUS v. 2015

(HOUGHTON MIFFLIN HARCOURT)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
K	<ul> <li>Teacher Edition (A &amp; B)</li> <li>Implementation Guide</li> <li>Assessment Package</li> <li>Enrichment Bundle</li> <li>Extra Practice Set</li> <li>Teacher and Student Activity Cards</li> <li>Home -to- School Connection Book</li> <li>Online Teacher Technology Kit</li> <li>Big Book Set</li> <li>Online Interactive Whiteboard Lessons</li> </ul>	<ul> <li>Student Edition A – Pt. 1</li> <li>Student Edition A – Pt. 2</li> <li>Student Edition B – Pt. 1</li> <li>Student Edition B – Pt. 2</li> <li>Online Student Technology Kit</li> </ul>
1	<ul> <li>Teacher Edition (A &amp; B)</li> <li>Implementation Guide</li> <li>Assessment Package</li> <li>Enrichment Bundle</li> <li>Extra Practice Guide</li> <li>Reteaching Guide</li> <li>Home -to- School Connection Book</li> <li>Online Teacher Technology Kit</li> <li>Fact Fluency</li> <li>Online Interactive Whiteboard Lessons</li> </ul>	<ul> <li>Student Texts (A &amp; B)</li> <li>Student Workbooks</li> <li>Online Student Technology Kit</li> <li>Student Interactivities</li> </ul>
2-5	<ul> <li>Teacher Edition (A &amp; B)</li> <li>Implementation Guide</li> <li>Assessment Package</li> <li>Enrichment Bundle</li> <li>Extra Practice Guide</li> <li>Transition Guides</li> <li>Reteaching Guide</li> <li>Home -to- School Connection Book</li> <li>Online Teacher Technology Kit</li> <li>Fact Fluency</li> <li>Online Interactive Whiteboard Lessons</li> </ul>	<ul> <li>Student Texts (A &amp; B)</li> <li>Student Workbooks</li> <li>Online Student Technology Kit</li> <li>Student Interactivities</li> </ul>

#### **Supplemental Resources**

#### **Great Minds**

https://greatminds.org/

#### **Embarc**

https://embarc.online/

#### **Engage NY**

http://www.engageny.org/video-library?f[0]=im\_field\_subject%3A19

#### **Common Core Tools**

http://commoncoretools.me/

http://www.ccsstoolbox.com/

http://www.achievethecore.org/steal-these-tools

#### **Achieve the Core**

http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

#### Manipulatives

http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project : <a href="http://illustrativemathematics.org/standards/k8">http://illustrativemathematics.org/standards/k8</a>

**Inside Mathematics:** http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education: <a href="https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx">https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx</a>

**Gates Foundations Tasks:** <a href="http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf">http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf</a>

## References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products